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Method for Measuring Damping about the Input Axis of a Single-Degree-of-Freedom Floated Gyro

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Nomenclature

D_i = damping about input axis
D_0 = damping about output axis
H = angular momentum
i = current into torquer
I_i = inertia about input axis
I_0 = inertia about output axis
K_0 = spring constant about axis
K_p = pickoff constant
K_T = torquer scale factor
T_A = torque applied = iK_T
T_D = restraint torques
ϕ = float rate about input axis
ψ = angular position of float about output axis
$\dot{\psi}$ = float rate about output axis
$\ddot{\psi}$ = float acceleration about output axis

WITH new and more sophisticated uses for high-precision gyroscopes, more critical examination of their behavior is required. For example, gyroscopic devices commonly are classified as either single-degree-of-freedom or two-degree-of-freedom units. In actuality, under fine analysis all gyroscopic devices exhibit two-degree-of-freedom performance. This is true for the single-degree-of-freedom gyro because of the clearance requirements in the pivot and jewel assembly that supports the structure containing the gyro rotor (Fig. 1). This clearance allows the float to rotate about its input axis, causing it to behave, under certain conditions, as a two-degree-of-freedom gyro.

This behavior exists only when the gyro senses a change in an applied rotation, and it ceases when the clearance is taken up. Although the effect is small since the angular clearance in a precision gyro is usually less than 1 min of arc, it can prove troublesome. It is of particular importance in inertial platform stabilization loops. Under the proper combination of small input angle excursions, it can result in improper stabilization due to gyro nutation and limit cycle effects.

Another area of concern is in the application of single-degree-of-freedom gyros to body-mounted guidance systems. Here, the gyro dynamics about the input axis produce a lag in the rate-measuring process, causing the introduction of cross-coupling errors. Thus, in order to evaluate and to simulate properly the behavior of a single-degree-of-freedom gyro in a sophisticated system, the dynamics of the gyro about its input axis must be known.

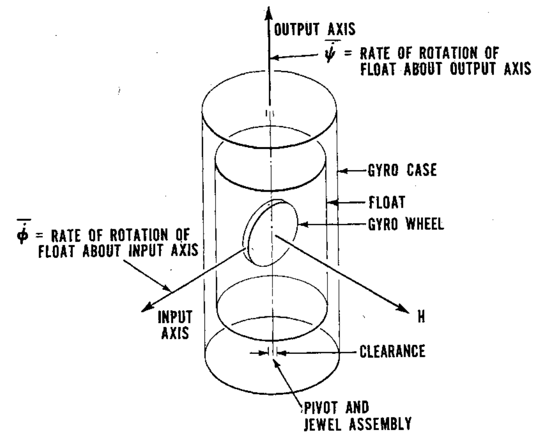

Fig. 1 Single-degree-of-freedom floated rate integrating gyro (actually, the float has movement about the input axis as well as about the output axis)

Two factors, inertia and damping, make up the input axis dynamics. The former can be calculated; the latter has been estimated by means of analytical computations but, to the author's knowledge, never has been measured experimentally. This article describes in detail an experimental procedure for measuring damping about the input axis of a single-degree-of-freedom floated gyro.

Figure 2 shows the equipment necessary to measure input axis damping. None of this equipment is itself novel; it is standard test equipment available wherever gyro evaluation is taking place. What is new is the method of using the equipment and the calculations shown here. The only information required is the output axis damping and the angular momentum of the gyro.

In this method, an accurately known current is put into the gyro torquer which sets up a known force on the float about the output axis, causing the latter to rotate. This can be described mathematically by the equation

$$iK_T = I_0\ddot{\psi} + D_0\dot{\psi} + K_0\psi \tag{1}$$

Since the spring constant in rate-integrating gyros is kept extremely small, Eq. (1) can be rewritten as

$$iK_T = T_A = I_0\ddot{\psi} + D_0\dot{\psi} \tag{2}$$

In Laplace notation

$$T_A(s) = (I_0S + D_0)\dot{\psi}(s) \tag{3}$$

Solving for the output axis rate,

$$\dot{\psi}(s) = \frac{T_A(s)}{I_0S + D_0} = \frac{T_A(s)/D_0}{(I_0/D_0)S + 1} \tag{4}$$

It should be noted that Eq. (4) holds true only when no pivot clearance exists. Actually, pivot clearances do exist,

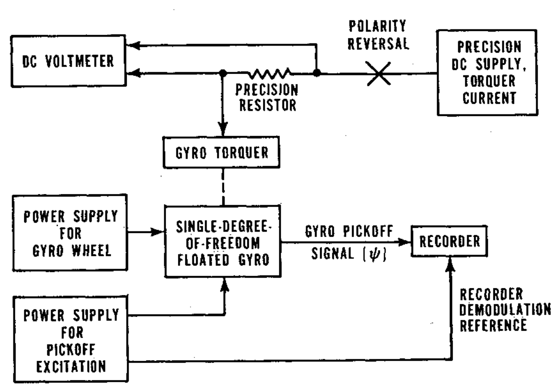

Fig. 2 Equipment necessary to measure damping about the input axis of a single-degree-of-freedom floated gyro

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and so the float rate $\dot{\psi}$ must be modified, giving rise to the method described herein.

Derivation

As can be seen from Eq. (4), an input current to the torquer causes a rate $\dot{\psi}$ about the output axis. With the gyro wheel operating, an angular momentum vector exists which reflects this rate as a torque applied about the input axis, as shown by

$$\dot{\boldsymbol{\psi}} \times \mathbf{H} = \mathbf{T}_i \qquad (5)\dagger$$

This torque, acting about the input axis, causes a rate that is derived in a similar fashion to that shown in Eqs. (1–4), yielding

$$\dot{\phi}(s) = T_i(s)/(I_iS + D_i) \qquad (6)$$

This rate in turn is reflected back onto the output axis by the gyroscopic equation, causing an opposing torque to the original torque applied. This opposing torque equals

$$\dot{\boldsymbol{\phi}} \times \mathbf{H} = \mathbf{T}_0 \qquad (7)$$

Summing torques about the output axis, one has

$$\Sigma\text{torques} = \mathbf{T}_A(s) - \mathbf{T}_0(s) \qquad (8)$$

Therefore, Eq. (4) must be modified accordingly:

$$(I_0S + D_0)\dot{\psi}(s) = T_A(s) - T_0(s) \qquad (9)$$

Using Eqs. (5–7), one finds that

$$\dot{\psi}(s) = \frac{D_i\, T_A(s)\,[(I_i/D_i)S + 1]}{D_iD_0[(I_0/D_0)S + 1][(I_i/D_i)S + 1] + H^2} \qquad (10)$$

From design calculations, the time constants I_0/D_0 and I_i/D_i can be shown to be negligible ($I_0D_0 = 5 \times 10^{-3}$ sec, and $I_i/D_i = 1 \times 10^{-5}$ sec). Therefore, for a step input of T_A, Eq. (10) then can be rewritten in the time domain:

$$\dot{\psi}_1 \approx D_iT_A/(D_iD_0 + H^2) \qquad (11)$$

Equation (11) describes the output rate during the time the float travels through the pivot clearance. In contrast, the rate in the time domain when the pivot clearance has been taken up is given from Eq. (4) as

$$\dot{\psi}_2 \approx T_A/D_0 \qquad (12)$$

The difference $\Delta\dot{\psi}$ between these two rates can be obtained by subtracting Eq. (11) from Eq. (12):

$$\Delta\dot{\psi} = \dot{\psi}_2 - \dot{\psi}_1 = H^2T_A/D_0(D_iD_0 + H^2) \qquad (13)$$

If this difference is measured, all the quantities in Eq. (13) are known except D_i, which is the damping about the input axis of the gyro. This quantity now can be solved for

$$D_i = (H^2T_A - \Delta\dot{\psi})/D_0{}^2\Delta\dot{\psi} \qquad (14)$$

Because of the existence of restraint torques (T_D) about the output axis of a gyro, the idealized representation given in Eq. (13) must be modified. Since these torques act in a manner identical to that of the applied torque T_A, Eq. (13) is rewritten to include this term as follows:

$$\Delta\dot{\psi} = \dot{\psi}_2 - \dot{\psi}_1 = H^2(T_A + T_D)/D_0(D_iD_0 + H^2) \qquad (15)$$

If the applied torque is reversed, that is, if the current applied to the gyro torquer has its polarity reversed, another relationship for $\Delta\dot{\psi}$ is defined:

$$\Delta\dot{\psi} = \dot{\psi}_2 - \dot{\psi}_1 = H^2(-T_A + T_D)/D_0(D_iD_0 + H^2) \qquad (16)$$

Note that T_D, being a fixed torque, does not change signs.

† Basic gyro equation in vector notation.

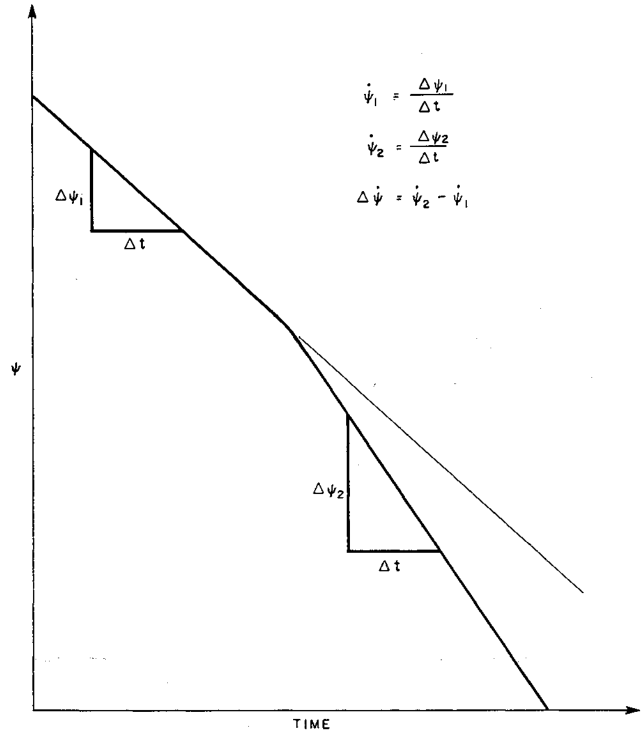

Fig. 3 Theoretical plot of gyro pickoff signal vs time; by means of equations here, the difference in slopes of the two sections of the curve gives input axis damping

Subtracting Eq. (16) from Eq. (15) and averaging yields

$$\Delta\dot{\psi} = H^2T_A/D_0(D_iD_0 + H^2) \qquad (17)$$

which is identical to the idealized Eq. (13). By this additional reversal, the effects of fixed restraint torques are eliminated. In the case where the unbalance torques are known, they can be biased out by a current applied to the gyro torquer in addition to the test current. Should this condition exist, the forementioned averaging technique would not be required.

If care is taken to eliminate restraint torques, the accuracy of the damping measurement then is limited by the gyro constants used in evaluating Eq. (14). It can be seen, for example, that an error in the knowledge of the gyro torquer constant would reflect directly as an error in the measurement of input axis damping. With present techniques in determining these constants, the damping measurement can be made with an accuracy of 5%.

Returning to Fig. 2, the test procedure now can be described. For convenience, the gyro is mounted in polar configuration (input axis sees no earth's rate). Current is applied to the gyro torquer through a precision resistor, enabling accurate measurement of the current. Knowing the torquer scale factor, the applied torque T_A can be calculated.

Torque is applied first in order to insure that the pivot clearance is taken up in one direction. The polarity then is reversed, forcing the float to rotate in the opposite direction. This latter rotation rate, which includes motion about both the input and output axes, was given as $\dot{\psi}_1$ in Eq. (11). The rate is recorded as the slope of the gyro pickoff signal on a recorder with a time reference (top portion of curve in Fig. 3).

When the pivot clearance is taken up, motion ceases about the input axis, and the rate changes to that given by $\dot{\psi}_2$ in Eq. (4). It is shown as the slope of the bottom portion of the curve in Fig. 3. The rate difference $\Delta\dot{\psi}$ thus is shown by the difference in the slopes of the two curves.

If restraint torquers are not biased, as shown in Fig. 2, an additional polarity reversal must be made, forcing the

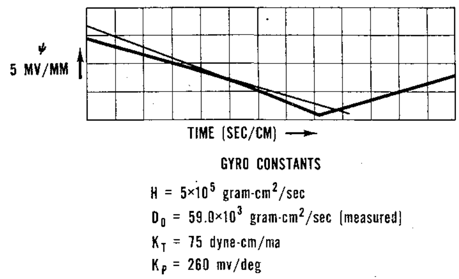

Fig. 4 Actual plot of gyro pickoff signal vs time

float to rotate in the opposite direction. With the use of Eq. (14), where $\Delta\dot\psi$ either is measured directly or is an average quantity, input axis damping now can be computed.

Figure 4 is an actual trace of a gyro pickoff taken during input axis damping tests on one of Kearfott's experimental gyros. At the time of this test, the gyro was mounted in a polar configuration with its fixed restraint torques biased out by a precision supply. To obtain the trace, a current of 1 ma was fed into the gyro torquer. The resulting rates $\dot\psi_1$ and $\dot\psi_2$ were taken from the curve as 0.055 and 0.073 deg/sec, respectively.

With the constants given in Fig. 4 and with the use of Eq. (14), input axis damping was measured as 1.3×10^7 g-cm^2/sec. This figure provided an experimentally derived damping ratio of $D_i/D_0 = 220$.

Impossibility of Linearizing a Hot-Wire Anemometer for Measurements in Turbulent Flows

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Turbulence investigations are having to deal increasingly with situations in which the fluctuations are not much smaller than the mean velocity. It is well known that, in these circumstances, the nonlinearity of a hot-wire anemometer gives rise to large errors. This had led to the use of systems in which the electronic circuit contains some element with a reciprocal nonlinearity so that the output is linear with the fluid velocity. The authors wish to point out that there is a basic difficulty in this procedure. (Doubtless many workers are aware of this, but, to the knowledge of the authors, it never has been stated explicitly, and one gets the impression that sometimes it is overlooked. References 1 and 3 contain some related remarks.)

The difficulty arises from the fact that a hot wire responds to the instantaneous velocity V normal to its axis,‡ and the relationship between this and a single velocity component ($V = [(U + u_1)^2 + u_2^2]^{1/2}$ in the case of a single wire normal to the mean stream) is a nonlinear one. Electronic correction may be devised for nonlinearities between V and the signal obtained, e.g., King's law, but it is inherently impossible to correct for this further effect as the correction depends on u_2, which is not being measured independently. When one attempts to measure it by using X wires, the u_3 component comes in in the the same way to introduce large errors when there are large fluctuations.

The errors arising from this effect are of the same order as those involved in nonlinear, e.g., constant-current, operation in the following sense. When one analyzes linear operation by the series-expansion method (along the lines of Ref. 2, pp. 97–100), terms of the same type and degree appear. However, the errors may be substantially different because of differences in the coefficients of these terms. This depends on the particular quantity that is being measured, and it is difficult to generalize about the merits of linearization. Furthermore, the relevance of conclusions, which are reached on the basis of the first few terms of series expansions, to the flows with large fluctuations for which the matter is particularly important, is open to question. However, errors so estimated at least may illustrate the situation. A few remarks on the measurement of different quantities will be presented below. The notation is as follows: U is the mean stream speed; u_1, u_2, and u_3 are the components of the velocity fluctuation, the first being taken in the mean stream direction; and $< >$ signifies a mean value.

Hinze[2] has used the example of isotropic, normally distributed, normally correlated turbulence with $<u_1^2>^{1/2}/U = 0.2$ for estimating the seriousness of errors in hot wire anemometry. It will be convenient to follow suit here. However, some of the terms in the expansions, in general, are not assessed readily, and it is not clear how typical this example is. This is usually particularly true of the terms, such as the first two terms of Eq. (2-44) of Ref. 2, that involve fluctuating velocities to one degree higher than the quantity being measured. (These will be referred to below as first-order terms.) These terms are zero in isotropic turbulence, and then the error is produced by the second-order terms, but they are not zero in general. Next, measurement of particular quantities is considered.

1. Mean Velocity Measured with a Single Wire

The expression given by a linear relationship between velocity and signal analogous to Eq. (2-40) of Ref. 2 is

$$U_{\text{meas}} = U_{\text{act}}[1 + (\tfrac{1}{2}<u_2^2>/U^2) + \ldots]$$

Hence, linearized operation is no better, and probably worse, than the straightforward constant-current operation for U measurement. This is the most drastic of the present conclusions. One would not, of course, construct linearizers just for mean-velocity measurements, but, having constructed them for turbulence measurements, one also might use them for measuring mean velocities and suppose that there was less error than usual. Perhaps it is not appreciated always that this supposition is fallacious.

2. $<u_1^2>$ Measured with a Single Wire

The expression for linearized operation corresponding to Eqs. (2-46) and (2-47) of Ref. 2 is

$$<u_1^2>_{\text{meas}} = <u_1^2>_{\text{act}}\left[1 + \frac{<u_1u_2^2>}{U<u_1^2>} + \frac{1}{4}\frac{<u_2^4>}{U^2<u_1^2>} - \frac{1}{4}\frac{<u_2^2>}{U^2<u_1^2>} - \frac{<u_1^2u_2^2>}{U^2<u_1^2>} + \ldots\right]$$

For isotropic, normally distributed, and normally correlated turbulence with 20% intensity, this indicates a 2% error, and linearization is markedly advantageous. Moreover, apart

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‡ The further complication of the effect of yaw is being ignored. There are circumstances in which this is not allowable,[1] but one can try to avoid those circumstances, whereas the point being discussed here always will arise.